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Comments on gluino condensates in $\mathcal{N} = 1/2$ SYM theory

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ABSTRACT: Using Ward identities of $\mathcal{N} = 1/2$ supersymmetric Yang-Mills theory, we show that while antichiral gluino condensates remain invariant under the C deformation, chiral gluino correlators can get contributions from all gauge fields with instanton numbers $k \leq 1$. In particular, a Ward identity of the $\mathrm{U}(1)_R$ symmetry allows us to determine the explicit dependence of chiral gluino correlators on the deformation parameter C.

KEYWORDS: Nonperturbative Effects, Solitons Monopoles and Instantons, Supersymm etric Effective Theories.

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Contents

1. Introduction

Four dimensional $\mathcal{N}=1$ supersymmetric gauge theories naturally arise as low energy effective field theories of D-branes when string theory is compactified over Calabi-Yau 3-folds. One could also turn on a graviphoton background, and look at the consequence it has on the effective theory of D-brane. The interesting observation is that in this background the odd coordinates of superspace become nonanticommuting over the brane [1]–[4]. This has the effect of reducing the number of supersymmetries to half. However, one can still construct a super Yang-Mills theory describing the effective theory on the brane [5]. Different aspects of this model have been further studied in [6]–[14]. While the generalization to $\mathcal{N}=2$, along with other interesting features of noncommutative superspace have further been explored in [15]–[24].

In this note we are to study the effect of C deformation on the gluino correlators. Using the Ward identity of the unbroken supersymmetry Q, we will see that the partition function as well as the antichiral gluino condensates remain invariant under the C deformation. Namely they will have the same value as in $\mathcal{N}=1$ SYM U(N) gauge theory, and in particular $\langle \mathrm{tr}(\overline{\lambda\lambda})^N\rangle$ gets contribution only from one instanton sector. In contrast, chiral correlation functions of gluini can get contributions not only from one anti-instantons ($F^-=0$), the only configurations which contribute to the chiral gluino correlators in $\mathcal{N}=1$ theory, but also from gauge fields of topological charges $k\leq 0$. In what follows, we will discuss the nature of these field configurations, and their possible contributions to the chiral correlators using Ward identities of the anomalous U$(1)_R$ symmetry. Although R symmetry is explicitly broken at the classical level, treating the deformation parameter C as a background field with an R charge the corresponding Ward identity tells us about the dependence of the correlators on the deformation parameter.

As (anti)instantons play an important role in our discussion of (anti)chiral correlators of gluini, let us briefly discuss how they might get deformed in the presence of fermions [12]. In $\mathcal{N}=1/2$ SYM model, the equation of motion for the gauge field read

$$D_\mu\left(F^{\mu\nu}+iC^{\mu\nu}\overline{\lambda\lambda}\right)=\sigma^\nu_{\alpha\dot\alpha}\,\{\lambda^\alpha\,,\overline{\lambda}^{\dot\alpha}\}\,, \tag{1.1}$$

while for $\overline{\lambda}$ and λ we have

$$\begin{aligned}\overline{\not{D}}\lambda &= -C^{\mu\nu}F^{+}_{\mu\nu}\overline{\lambda} - i\frac{|C|^2}{2}(\overline{\lambda}\overline{\lambda})\overline{\lambda} \\ \not{D}\overline{\lambda} &= 0\,.\end{aligned} \tag{1.2}$$

Note that when $F^{-}_{\mu\nu} = 0$, the last equation has no solution. Setting $\overline{\lambda} = 0$, then $F^{-}_{\mu\nu} = 0$ solves the first equation, while the second equation reduces to the ordinary zero mode equation for chiral fermions. Therefore anti-instantons ($F^{-} = 0$) and the corresponding chiral zero modes are a solution to the above field equations. What about instantons ($F^{+} = 0$)? In this background there are antichiral zero modes $\overline{\lambda}$, and hence the second equation implies that chiral fermions λ cannot be zero either. On the other hand, using the Bianchi identity in the first equation we have

$$iC^{\mu\nu}D_{\mu}\left(\overline{\lambda\lambda}\right) = \sigma^{\nu}_{\alpha\dot{\alpha}}\,\{\lambda^{\alpha}\,,\overline{\lambda}^{\dot{\alpha}}\}\,. \tag{1.3}$$

Acting now with the covariant derivative D_{ν} on the above equation, and using $F^{+} = 0$ implies that $\overline{\not{D}}\lambda = 0$, which is inconsistent with the first equation of (1.2). So we conclude that instantons cannot be a solution to the equations (1.1) and (1.2). In [12] it was argued that instanton like solutions are still possible if we instead consider the deformed instanton equations

$$\begin{aligned}F^{+}_{\mu\nu} + \frac{i}{2}C_{\mu\nu}\overline{\lambda\lambda} &= 0 \\ \not{D}\overline{\lambda} &= 0 \\ \overline{\not{D}}\lambda &= 0\,,\end{aligned} \tag{1.4}$$

which like instantons saturate a BPS bound with an action $\mathcal{S} = \frac{-8\pi^2 k}{g^2}$, $k < 0$, where k is the instanton number. Since $k < 0$, the index theorem implies that $\overline{\lambda}$ cannot be zero, and for the first equation above to be consistent with eq. (1.1) we must have $\lambda = 0$. The solutions to the above equations, with $\lambda = 0$, now solve the field equations (1.1) and (1.2). In fact we will see that a C dependent contribution to the chiral gluino correlators arises exactly because of the existence of these classical field configurations. Below, we will examine the possible contributions of the above classical field configurations on both chiral and antichiral gluino correlators. Notice that both configurations above, i.e., anti-instantons and those of (1.4), are supplemented with the ordinary Dirac equation for chiral and antichiral fermions. As discussed in [12], these are also the corresponding equations for the fermionic zero modes. Therefore, in a perturbative expansion around these solutions, one needs to take care of the zero modes of the Dirac operator by inserting appropriate operators in the path integral, just as in $\mathcal{N} = 1$ SYM theory.

2. Antichiral gluino condensates

Let us begin with assuming that the superspace coordinates θ^{α} are not anticommuting, and instead they satisfy the following anticommutation relation

$$\{\theta^{\alpha}, \theta^{\beta}\} = C^{\alpha\beta}\,, \tag{2.1}$$

where $C^{\alpha\beta}$ is a constant and symmetric 2×2 matrix. This deformation of the superspace has been studied earlier in [25, 26, 3]. The anticommutation relation (2.1) will deform the supersymmetry algebra with $\overline{Q}^2$ proportional to the deformation parameter $C^{\alpha\beta}$. Seiberg [5] has considered the above deformation in $\mathcal{N}=1$ supersymmetric model and has shown that half of the supersymmetries can be preserved. Indeed if $W^\alpha = (A_\mu, \lambda)$ denotes the usual $\mathcal{N}=1$ gauge super multiplet, then the lagrangian of this $\mathcal{N}=1/2$ model reads

$$\begin{aligned} \mathcal{L} = & \frac{i\tau}{16\pi}\int d^2\theta \, \mathrm{tr}\, W^\alpha W_\alpha - \frac{i\bar{\tau}}{16\pi}\int d^2\bar{\theta}\, \mathrm{tr}\, \overline{W}^{\bar{\alpha}}\overline{W}_{\bar{\alpha}} \\ & + \frac{(i\tau - i\bar{\tau})}{16\pi}\left(-iC^{\mu\nu}\, \mathrm{tr}\, F_{\mu\nu}\overline{\lambda}\overline{\lambda} + \frac{|C|^2}{4}\, \mathrm{tr}(\overline{\lambda}\overline{\lambda})^2\right), \end{aligned} \tag{2.2}$$

where

$$C^{\mu\nu} \equiv C^{\alpha\beta}\epsilon_{\beta\gamma}\sigma^{\mu\nu}_{\alpha}{}^{\gamma}$$

is a constant and antisymmetric self-dual matrix, with $|C|^2 = C_{\mu\nu}C^{\mu\nu}$, and τ is the complex coupling constant

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}. \tag{2.3}$$

The above lagrangian is invariant under the following Q deformed supersymmetry transformations,

$$\begin{aligned} \delta\lambda &= i\epsilon D + \sigma^{\mu\nu}\epsilon\left(F_{\mu\nu} + \frac{i}{2}C_{\mu\nu}\overline{\lambda}\overline{\lambda}\right) \\ \delta A_\mu &= -i\overline{\lambda}\bar{\sigma}_\mu\epsilon \\ \delta D &= -\epsilon\sigma^\mu D_\mu\overline{\lambda} \\ \delta\overline{\lambda} &= 0, \end{aligned} \tag{2.4}$$

whereas $\overline{Q}$ is broken. Solving for the auxillary field D, the action reads

$$\mathcal{S} = \frac{1}{2g^2}\int d^4x\ \mathrm{tr}\left(\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + 2i\lambda\, \not{D}\overline{\lambda} + iC^{\mu\nu}F_{\mu\nu}\overline{\lambda}\overline{\lambda} - \frac{1}{4}|C|^2\,(\overline{\lambda}\overline{\lambda})^2\right). \tag{2.5}$$

An interesting observation is that while the terms proportional to the deformation parameter $C^{\mu\nu}$ cannot be written as a Q commutator, the variation of the lagrangian with respect to $C^{\mu\nu}$ can:

$$\begin{aligned} \delta\mathcal{L} &= \frac{i}{2g^2}\delta C^{\mu\nu}\, \mathrm{tr}\left(F_{\mu\nu}\overline{\lambda}\overline{\lambda} + \frac{i}{2}C_{\mu\nu}\,(\overline{\lambda}\overline{\lambda})^2\right) \\ &= -\frac{i}{4g^2}\left\{Q^\alpha\,,\, \delta C^{\mu\nu}(\sigma_{\mu\nu})_{\alpha\beta}\ \mathrm{tr}(\lambda^\beta\overline{\lambda}\overline{\lambda})\right\}. \end{aligned} \tag{2.6}$$

Assuming supersymmetry is not spontaneously broken, i.e., $Q\,|0\rangle = 0$, this will have an important consequence that the partition function does not depend on C:

$$\frac{\delta Z}{\delta C^{\mu\nu}} = -\left\langle\frac{\delta\mathcal{S}}{\delta C^{\mu\nu}}\right\rangle = \frac{i}{4g^2}\int d^4x\left\langle\left\{Q^\alpha\,,\,(\sigma_{\mu\nu})_{\alpha\beta}\ \mathrm{tr}(\lambda^\beta\overline{\lambda}\overline{\lambda}(x))\right\}\right\rangle = 0. \tag{2.7}$$

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This even further extends to correlation functions of Q invariant operators. Let $\mathcal{O}^i$, for $i = 1, 2, \ldots, n$, be operators such that $[Q, \mathcal{O}^i\} = 0$, then we will have

$$\begin{aligned}
\frac{\delta}{\delta C^{\mu\nu}} \langle \mathcal{O}^1 \mathcal{O}^2 \ldots \mathcal{O}^n \rangle =& \frac{i}{4g^2} \int d^4x \left\langle \mathcal{O}^1 \mathcal{O}^2 \ldots \mathcal{O}^n \left\{ Q^\alpha , (\sigma_{\mu\nu})_{\alpha\beta} \operatorname{tr}(\lambda^\beta \overline{\lambda\lambda}(x)) \right\} \right\rangle \\
=& \frac{i}{4g^2} \int d^4x \left\langle \mathcal{O}^1 \mathcal{O}^2 \ldots \mathcal{O}^n \, Q^\alpha \, (\sigma_{\mu\nu})_{\alpha\beta} \operatorname{tr}(\lambda^\beta \overline{\lambda\lambda}(x)) \right\rangle \\
=& \pm \frac{i}{4g^2} \int d^4x \left\langle \, Q^\alpha \mathcal{O}^1 \mathcal{O}^2 \ldots \mathcal{O}^n (\sigma_{\mu\nu})_{\alpha\beta} \operatorname{tr}(\lambda^\beta \overline{\lambda\lambda}) \right\rangle = 0 \, .
\end{aligned} \tag{2.8}$$

Therefore, in computing the correlation functions of Q invariant operators one can choose a convenient value of C. One such a choice is set C to zero reducing the lagrangian to that of $\mathcal{N} = 1$ SYM theory. Hence, such operators will have the same expectation value as in pure $\mathcal{N} = 1$ SYM theory. In particular, the antichiral gluino correlator will be invariant under the C deformation:

$$\frac{\delta}{\delta C^{\mu\nu}} \langle \operatorname{tr} \overline{\lambda\lambda}(x_1) \; \operatorname{tr} \overline{\lambda\lambda}(x_2) \ldots \operatorname{tr} \overline{\lambda\lambda}(x_N) \rangle = 0 \, . \tag{2.9}$$

Moreover, one can use the Ward identity of the unbroken Q supersymmetry to show that the above correlation function is x-independent. Cluster decomposition then implies that the operator $\operatorname{tr}(\overline{\lambda\lambda})$ gets a C independent vacuum expectation value just as in pure $\mathcal{N} = 1$ SYM theory [27]:

$$\langle \operatorname{tr}(\overline{\lambda\lambda}) \rangle = A \bar{\Lambda}^3 \, , \tag{2.10}$$

with the one-loop Renormalization Group (RG) invariant scale, $\bar{\Lambda}$, related to the microscopic cutoff Λ_0 through

$$\bar{\Lambda}^{3N} = \Lambda_0^{3N} \, e^{-2\pi i \overline{\tau}} \, . \tag{2.11}$$

3. Chiral sector

What happens to chiral operators such as $\operatorname{tr}(\lambda\lambda)$? To see the consequence of C-deformation on the correlation functions of such operators we use a Ward identity corresponding to the $\mathrm{U}(1)_R$ symmetry. To start with, let us first discuss the case of $\mathcal{N} = 1$ SYM theory in some more detail. Classically there is a $\mathrm{U}(1)_R$ symmetry acting on the fields as

$$\lambda \to e^{i\alpha} \lambda \, , \qquad \overline{\lambda} \to e^{-i\alpha} \overline{\lambda} \, . \tag{3.1}$$

Quantum mechanically, however, $\mathrm{U}(1)_R$ is reduced to Z_{2N}. Roughly speaking, this is seen by noticng that in the background of a gauge field with the topological charge k, the number of chiral and antichiral zero modes , n_+ and n_- respectively, are related to the index of the Dirac operator in that background as follows

$$n_+ - n_- = \text{index} \, \not{D} = 2kN \, , \tag{3.2}$$

and thus under $\mathrm{U}(1)_R$ the measure of the path integral in this background transforms as

$$\mathcal{D}\overline{\lambda}\, \mathcal{D}\lambda \to e^{-2iNk\alpha} \, \mathcal{D}\overline{\lambda}\, \mathcal{D}\lambda \, , \tag{3.3}$$

whereas for nonzero modes the measure is invariant. Notice that in writing the measure we use the eigenfunctions of the Dirac operator as a basis, and that the above index theorem holds for any gauge field which has a topological charge k, independent of the equation it satisfies. Clearly (3.3) shows that the subgroup Z_{2N} is left unbroken. This latter symmetry is further spontaneously broken to Z_2 by gluino condensations. Chiral gluino condensation happens because of the existence of anti-instantons with $k = 1$. In this background there are $2N$ chiral zero modes, and to saturate them one inserts in the path integral N gauge invariant operators made up of chiral fermions $\mathrm{tr}(\lambda\lambda)$ of the form

$$G(x_1, \ldots, x_N) = \langle \mathrm{tr}\, \lambda\lambda(x_1)\ \mathrm{tr}\, \lambda\lambda(x_2) \ldots \mathrm{tr}\, \lambda\lambda(x_N)\rangle\,. \tag{3.4}$$

In pure $\mathcal{N} = 1$ SYM theory, one can also use the Ward identities of $\overline{Q}$ symmetry (of the kind we used in eq. (2.8)) to show that the above correlation function does not depend on x_i's. Using this and cluster decomposition one then concludes that $\mathrm{tr}\, \lambda\lambda$ gets a vacuum expectation value, and hence a spontaneous breaking of Z_{2N} to Z_2 leaving N distinct quantum vacuua. Hence, we see a very similar behaviour between the chiral and antichiral sectors of $\mathcal{N} = 1$ theory.

The situation for $\mathcal{N} = 1/2$ SYM theory is though quite different. In fact, since there is no $\overline{Q}$ supersymmetry, the whole argument above including the x-independence of $G(x_1, \ldots, x_N)$, and the subsequent spontaneous breaking of Z_{2N} to Z_2 breaks down. So the lack of $\overline{Q}$ supersymmetry forbids us from going any further in this direction. However, we can still use the anomalous $\mathrm{U}(1)_R$ symmetry to discuss the possible C dependence of correlation functions like (3.4). Before proceeding in the anomaly discussion note that, since anti-instantons ($F^- = 0$) and the corresponding chiral zero modes ($\overline{\not{D}}\lambda = 0$) are solutions to the equations of motion, the operators inserted in (3.4) continue to saturate the $2N$ chiral zero modes of the Dirac operator in $k = 1$ topological sector. Gauge fields of higher topological charge ($k > 1$) give zero contribution because of the unsaturated zero modes ($n_+ = 2kN + n_-$). But, unlike pure $\mathcal{N} = 1$ SYM theory, gauge fields with $k < 0$ can also contribute to this correlation function. This happens, as we will see in the following, because the antichiral zero modes can be saturated by the C dependent terms already present in the action.

To obtain the Ward identity related to $\mathrm{U}(1)_R$ symmetry, first notice that the terms proportional to C in the action explicitly break the $\mathrm{U}(1)_R$ symmetry:

$$\delta_R \mathcal{S} = \frac{\alpha}{2g^2} \int d^4x\ \mathrm{tr}\big(2C^{\mu\nu} F_{\mu\nu} \overline{\lambda\lambda} + i\, |C|^2\, (\overline{\lambda\lambda})^2\big)\ , \tag{3.5}$$

for α an infinitesimal parameter of $\mathrm{U}(1)_R$. On the other hand, in the background of gauge fields with topological charge k, which we expand around, according to (3.3) the measure has a charge $-2kN$ under the $\mathrm{U}(1)_R$ symmetry. Suppose now that an operator $\mathcal{O}$ with an R charge equal to q is inserted in the path integral. The invariance of the whole path integral, with the operators inserted in, under a change of field variables results to the following Ward identity

$$\langle \mathcal{O}\, \delta_R \mathcal{S}\rangle = i\alpha\,(q - 2Nk)\langle \mathcal{O}\rangle\,. \tag{3.6}$$

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The above identity can be written in a more useful way. Using (2.6), we have

$$\delta_R \mathcal{S} = -2i\alpha\, C^{\mu\nu} \frac{\delta \mathcal{S}}{\delta C^{\mu\nu}}\,, \tag{3.7}$$

so the Ward identity (3.6) reads

$$C^{\mu\nu} \frac{\delta}{\delta C^{\mu\nu}} \langle \mathcal{O} \rangle_k = \frac{1}{2}(q - 2Nk)\langle \mathcal{O} \rangle_k\,, \tag{3.8}$$

where we have now put the subscript k to indicate the corresponding topological sector. Let us first discuss the case with $k > 0$. Taking $\mathcal{O}$ the operator inserted in (3.4) with $q = 2N$, and choosing $k = 1$, for instance, Ward identity (3.6) becomes

$$\langle \mathrm{tr}\,\lambda\lambda(x_1)\ \mathrm{tr}\,\lambda\lambda(x_2) \ldots \mathrm{tr}\,\lambda\lambda(x_N)\, \delta_R \mathcal{S} \rangle_{k=1} = 0\,, \tag{3.9}$$

or equivalently

$$\frac{\delta}{\delta C} \langle \mathrm{tr}\,\lambda\lambda(x_1)\ \mathrm{tr}\,\lambda\lambda(x_2) \ldots \mathrm{tr}\,\lambda\lambda(x_N) \rangle_{k=1} = 0\,, \tag{3.10}$$

where we have set

$$C_{12} = C_{34} \equiv \frac{C}{4}\,. \tag{3.11}$$

So for $k = 1$ we can again take the limit where C vanishes. This will reduce the computation to that of $\mathcal{N} = 1$ theory, where we know only $k = 1$ contributes to (3.4) [27]. Moreover, as said before, gauge fields of higher topological charge, $k > 1$, do not contribute to (3.4), hence we will have

$$\sum_{k\geq 1} G_k(x_1, \ldots, x_N)\, e^{2\pi i k\, \tau} = \langle \mathrm{tr}\,\lambda\lambda(x_1)\ \mathrm{tr}\,\lambda\lambda(x_2) \ldots \mathrm{tr}\,\lambda\lambda(x_N) \rangle_{C=0} = A \Lambda^{3N}\,. \tag{3.12}$$

For $k \leq 0$, and $q = 2N$, Ward identity (3.8) reads

$$C \frac{\delta}{\delta C}\, G_k(x_1, \ldots, x_N) = (N + N|k|)\, G_k(x_1, \ldots, x_N) \quad k \leq 0\,, \tag{3.13}$$

which, upon integration, results to the following C dependence of (3.4)

$$G_k(x_1, \ldots, x_N) \equiv \langle \mathrm{tr}\,\lambda\lambda(x_1)\ \mathrm{tr}\,\lambda\lambda(x_2) \ldots \mathrm{tr}\,\lambda\lambda(x_N) \rangle_k = A_k\, C^{N+N|k|}\,, \quad k \leq 0\,, \tag{3.14}$$

where A_k is in general a function of the coupling g^2, and the ultraviolet cutoff Λ_0. Also note that, as there is no $\overline{Q}$ symmetry, A_k might well depend on $|x_i - x_j|$. A question that may now arise is what are the classical field configurations which give rise to such a C dependent contributions? To answer this question let us look back at eqs. (1.4) and consider a perturbative calculation around the corresponding solutions. As discussed in Introduction, in this background there are no chiral zero modes, and according to the index theorem there are $2|k|N$ antichiral zero modes of the Dirac operator. To saturate these zero modes, we need to pull down $2|k|N$ terms proportional to $\overline{\lambda}$ from the action. Moreover, as there are no chiral zero modes, $2N$ chiral fermions in the correlator must get Wick contracted with additional $2N$ antichiral fermions brought down from the action. Noticing that each factor of $\overline{\lambda\lambda}$ comes with one power of C, then explains the factor of $C^{N+N|k|}$ appeared in (3.14). For the case of U(2) gauge group, and as a schematic illustration of the above perturbative calculation, in figures 1–4 we have depicted the related Feynman graphs to the lowest order in g^2. The subscript 0 there indicates the zero modes (fermionic classical field configurations of (1.4)).

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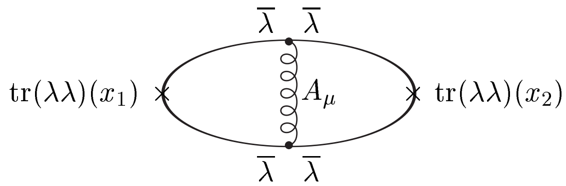

Figure 1: 2-loop diagram in $k = 0$ sector proportional to C^2.

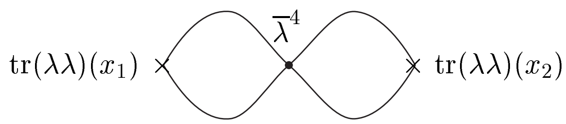

Figure 2: 2-loop diagram in $k = 0$ sector proportional to C^2.

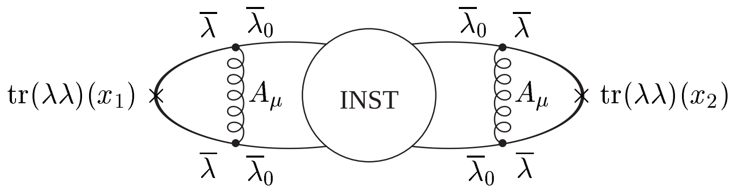

Figure 3: 2-loop diagram in $k = -1$ sector proportional to C^4.

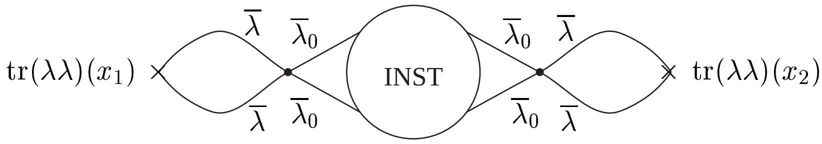

Figure 4: 2-loop diagram in $k = -1$ sector proportional to C^4.

To sum up the contributions from all topological sectors characterized by the instanton number k, we still need to take into account the θ angle contribution by multiplying each term in (3.14) by a factor of $e^{i\theta k}$. Moreover, since $\mathcal{S} = -8\pi^2 k/g^2$ in the background of (1.4), the weight factor adds up to $e^{2\pi i k\overline{\tau}}$. After all, we arrive at

$$\begin{aligned} G(x_1,\ldots,x_N) &= A\Lambda^{3N} + \sum_{k\leq 0} G_k(x_1,\ldots,x_N)\, e^{2\pi i k\overline{\tau}} \\ &= A\Lambda_0^{3N}\, e^{2\pi i\tau} + \sum_{k\leq 0} A_k\, C^{N+N|k|}\, e^{2\pi i k\overline{\tau}}\,, \end{aligned} \tag{3.15}$$

where we have taken into account the term $A\Lambda^{3N} = A\Lambda_0^{3N}\, e^{2\pi i\tau}$ in (3.12). Notice that the above expression for $G(x_1,\ldots,x_N)$ has the right R charge. In fact if we regard τ ($\overline{\tau}$) and C as background fields and let them transform under the $\mathrm{U}(1)_R$ group as follows

$$\begin{aligned} \tau &\rightarrow \tau + \frac{\alpha N}{\pi} \\ \overline{\tau} &\rightarrow \overline{\tau} + \frac{\alpha N}{\pi} \\ C &\rightarrow e^{2i\alpha}\, C\,, \end{aligned} \tag{3.16}$$

then by looking at (3.1), (3.3), and the action (2.5) we can see that the full path integral (either in the background of anti-instantons or generalized instantons (1.4)) is invariant under the R-symmetry group. Therefore, the expectation value of any operator must transform under the R-symmetry group exactly as the operator itself does. This is the behavior we observe in (3.15).

Two further comments about our result in (3.15) are in order. First note that the holomorphicity in τ of the chiral gluino correlators observed in $\mathcal{N} = 1$ theory, is now spoilt by the appearance of $\overline{\tau}$ in C dependent terms. The second comment concerns the calculation of A_k's in the above correlation function. Upon taking the $N + N|k|$ derivative of (3.14) with respect to C, we have

$$\frac{1}{(N+N|k|)!}\left(\frac{\delta}{\delta C}\right)^{N+N|k|} G_k(x_1,\ldots,x_N) = A_k\,. \tag{3.17}$$

As the right hand side is independent of C, we can take the limit of vanishing C in the left hand side to reduce it to a specific correlation function in $\mathcal{N} = 1$ SYM theory. In this way the computation of A_k's can be reduced to a computation in $\mathcal{N} = 1$ SYM theory.

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